

① Identify critical points:

$$\nabla f = \vec{0} \quad (\text{or DNE})$$

$$\begin{cases} 2x = 0 \\ 0 = 0 \end{cases}$$

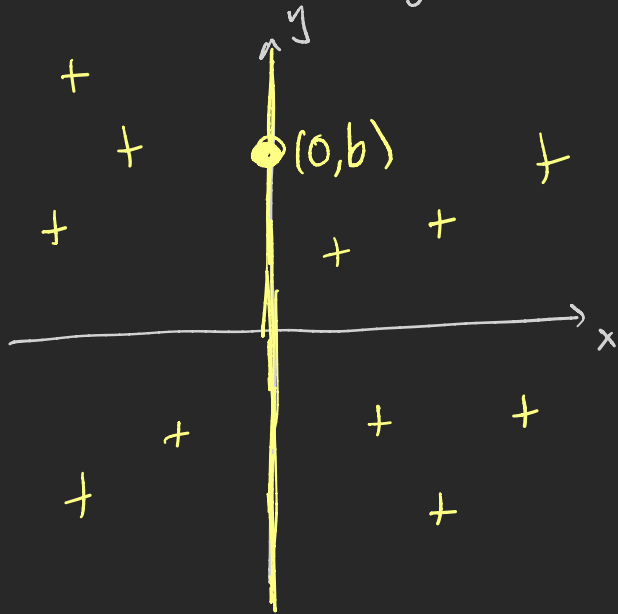
So the critical pts are all pts of the form $(0, b)$, i.e. the y -axis.

② Classify them:

$$\begin{aligned} \det H_f(0, b) &= \det \begin{bmatrix} f_{xx}(0, b) & f_{xy}(0, b) \\ f_{yx}(0, b) & f_{yy}(0, b) \end{bmatrix} \\ &\stackrel{D}{=} \det \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} = 0. \end{aligned}$$

$D=0 \Rightarrow$ 2nd deriv test is inconclusive \therefore

Need to resort to adhoc analysis of f :



To figure out whether (a, b) is an absolute min:

"Is it true that $f(x, y)$ is always at least as big as $f(a, b)$?"

In our example: $f(0, b) = 0^2 = 0$.

$f(x, y) = x^2 \geq 0$ so YES.

(An absolute min is automatically also a local min.)

To figure out whether (a, b) is a local min:

"Can I draw a circle around (a, b) so that, within that circle, $f(x, y)$ is always at least as big as $f(a, b)$?"